INTERFUEL SUBSTITUTION AND THE INDUSTRIAL DEMAND FOR ENERGY: AN INTERNATIONAL COMPARISON

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THE effects of GNP growth and changing fuel prices on the industrial demand for energy depend on the substitutability of energy and other factors of production and on the substitutability of fuels within the energy aggregate. The role of energy in the structure of production has been the focus of a number of recent studies, but the evidence on factor and fuel substitutability is mixed. Berndt and Wood (1975), Hudson and Jorgenson (1974), Fuss (1977), and Magnus (1979) all worked with data for a single country, and found energy and labor to be substitutes, but energy and capital to be *complements*. Griffin and Gregory (1976), using cross-section data at five-year intervals for nine countries to capture long-run effects, found energy and capital to be substitutes. Fuss (1977) found for Canada moderate substitutability among coal, gas and oil, but almost none between these fuels and electricity, while Halvorsen (1976) found greater substitutability among all fuels in the United States.

This paper provides another look at the role of energy in the structure of production. Like Griffin and Gregory we use international data to measure long-run effects, but we work with annual data to determine, where possible, international differences in elasticities, and we include individual fuels in the model. Our objectives are to provide some new evidence on the extent of capital, labor and energy substitutability, the long-run own and cross price elasticities of energy and individual fuels, the impact of growth in industrial activity on the demands for energy and individual fuels, and the effects, for different countries, of increased energy costs on the cost of output.

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Our approach, similar to that used recently by Fuss (1977) for Canadian data, is to estimate a translog cost function that is homothetically separable in the capital, labor and energy aggregates. This is consistent with producers choosing cost-minimizing factor inputs in two stages; energy costs are minimized in the choice of fuel inputs, and total costs are minimized in the choice of capital, labor and energy inputs. This also gives us increased degrees of freedom by allowing us to estimate, in stages, a homothetic cost function that aggregates fuel prices and generates an energy price index, and a non-homothetic cost function for the capital, labor and energy aggregates. On the other hand, it imposes restrictions on the structure of production, namely, that expenditure shares for fuels are independent of the expenditure shares for capital and labor, and expenditure shares for fuels are independent of total energy expenditures.

I. The Model

Our model requires certain restrictive assumptions about the structure of production. First, we assume that capital, labor, and energy inputs are as a group weakly separable from the fourth input, materials.[1] This assumption is necessary since we have no data from which to construct price indices of materials inputs, and therefore we can only estimate unrestricted elasticities of substitution among capital, labor, and energy. Second, we assume that the production function is weakly separable in the major categories of capital, labor, and energy.[2] This implies that the

[1] Materials includes intermediate inputs as well as non-energy raw materials. Weak separability here means that the marginal rate of substitution between any two of the first three inputs is independent of the quantity of materials used as an input. This is a necessary and sufficient condition for the production function to be of the form $Q = F[f(K,L,E); M]$. For a proof and further discussion, see Berndt and Christensen (1973).

[2] Halvorsen and Ford (1978) recently used translog cost functions to test for separability of the energy aggregate for each of eight individual two-digit industries in the United States. They found separability to hold for four of the eight industries.

marginal rates of substitution between individual fuels are independent of the quantities of capital and labor. The assumption permits us to use aggregate price indices for capital, labor, and energy inputs; in particular, to construct an energy price index that aggregates the prices of the four fuels, and to construct a price index of capital services that aggregates different types of capital. Finally, we assume that the capital, labor, and energy aggregates are homothetic in their components—in particular, that the energy aggregate is homothetic in its oil, gas, coal, and electricity inputs.[3] Denny and Fuss (1977) show that this last assumption provides a necessary and sufficient condition for an underlying two-stage optimization process, i.e., optimize the mix of fuels that make up the energy input, and then optimally choose quantities of capital, labor, and energy.

We can equivalently express these three assumptions by writing the production function as

$$Q = [(f(K,L,e(F_1,F_2,F_3,F_4));M], \tag{1}$$

where e is a homothetic function of the four fuels. If the factor prices and output level are exogenously determined, this production structure can alternatively be described by a cost function that is also weakly separable, i.e., a function of the form

$$C = G[g(P_K,P_L,P_E(P_{F1},P_{F2},P_{F3},P_{F4}),Q);P_M,Q]. \tag{2}$$

Here P_E is an aggregate price index of energy, i.e., a function that aggregates the fuel prices P_{Fi}. This "aggregator function" is homothetic, and thus does not include the total quantity of energy as one of its arguments.

Equation (2) can be characterized and estimated in stages. We first represent the price of energy (which is the unit cost of energy to a producer choosing fuel inputs) by a homothetic translog cost function with constant returns to scale. Estimation of the share equations implied by this cost function gives us the own and cross partial price elasticities for the four fuels, and the cost function itself provides an instrumental variable for the price of energy. Next we represent the cost of industrial output by a non-homothetic translog cost function. Estimation of the share equations implied by this cost function gives us demand and substitution elasticities for capital, labor, and energy.[4]

We review the properties of the translog cost function only briefly.[5] It is a second-order approximation to an arbitrary cost function, and has the form

$$\begin{aligned}\log C = {} & \alpha_0 + \alpha_Q \log Q + \sum_i \alpha_i \log P_i \\ & + \tfrac{1}{2}\gamma_{QQ}(\log Q)^2 \\ & + \tfrac{1}{2}\sum_i \sum_j \gamma_{ij}\log P_i \log P_j \\ & + \sum_i \gamma_{Qi}\log Q \log P_i \end{aligned} \tag{3}$$

where C is total cost, Q is output, and P_i are factor prices. From Shephard's Lemma, the derived demand functions are found by differentiating the cost function with respect to the prices, so that the share equations are given by $S_i = \partial \log C/\partial \log P_i = P_iX_i/C$, or

$$S_i = \alpha_i + \gamma_{Qi}\log Q + \sum_j \gamma_{ij}\log P_j, \quad i = 1, \ldots, n. \tag{4}$$

Since the shares must add to 1, only $n - 1$ of the share equations are estimated. Note that the parameters α_0, α_Q, and γ_{QQ} are not identified unless the cost function itself is estimated.

The cost function must be homogeneous of degree 1 in prices, and satisfy the conditions corresponding to a well-behaved production function. This implies the following parameter restrictions:[6] $\Sigma_i\alpha_i = 1$, $\Sigma_i\gamma_{Qi} = 0$, $\gamma_{ij} = \gamma_{ji}$, $i \neq j$, and $\Sigma_i\gamma_{ij} = \Sigma_j\gamma_{ij} = 0$. The cost function as specified is non-homothetic, and may have non-constant returns to scale. The cost function would be homothetic if it could be written as a separable function of output and factor prices. Thus the additional parameter restrictions $\gamma_{Qi} = 0$

[3] The second and third assumptions together are referred to as homothetic separability.

[4] We might have chosen to use translog production functions rather than cost functions in estimating elasticities. Since the translog cost and production functions are not self-dual, different elasticity estimates would result, and as Burgess (1975) has recently shown, the difference could be significant. However, we use the cost function since it is more appropriate to take prices as exogenous than quantities.

[5] The translog production function and cost function were introduced by Christensen, Jorgenson, and Lau (1973). Applications can be found in the work of Berndt and Christensen (1973), Berndt and Wood (1975), Christensen and Greene (1976), Fuss and Waverman (1975), Fuss (1977), Griffin and Gregory (1976), Halvorsen (1976), Hudson and Jorgenson (1974), Humphrey and Moroney (1975), and Moroney and Toevs (1977, 1978).

[6] See Christensen, Jorgenson, and Lau (1973).

must be added to impose homotheticity. The cost function is also homogeneous if the elasticity of cost with respect to output is constant, and this implies the additional restriction $\gamma_{QQ} = 0$. Finally, if the elasticities of substitution between all factors are equal to 1 (so that the cost function corresponds to a Cobb-Douglas production function), the additional parameter restrictions $\gamma_{ij} = 0$ are implied.

These restrictions can be tested using a simple chi-square test. An appropriate statistic is

$$-2 \log \Lambda = N(\log|\hat{\Omega}_r| - \log|\hat{\Omega}_u|) \quad (5)$$

where $|\hat{\Omega}_r|$ and $|\hat{\Omega}_u|$ are the determinants of the estimated error covariance matrices for the restricted and unrestricted models, and N is the number of observations. This statistic is distributed as chi-square with degrees of freedom equal to the number of parameter restrictions being tested.

Uzawa (1962) showed that the Allen partial elasticities of substitution can be computed from $\sigma_{ij} = CC_{ij}/C_iC_j$, so for the translog cost function we have

$$\begin{aligned} \sigma_{ij} &= (\gamma_{ij} + S_iS_j)/S_iS_j, \quad i \neq j \\ \sigma_{ii} &= [\gamma_{ii} + S_i(S_i - 1)]/S_i^2. \end{aligned} \quad (6)$$

The own and cross price elasticities of demand are thus given by $\eta_{ii} = \sigma_{ii}S_i$, and $\eta_{ij} = \sigma_{ij}S_j$.[7] However, these are *partial* price elasticities when applied to fuels; i.e., they account only for substitution between fuels, under the constraint that the total quantity of energy consumed remains constant. The *total* own price elasticity for each fuel $\eta^*_{ii} = d \log X_i/d \log P_i$ is given by

$$\eta^*_{ii} = \frac{P_i}{X_i}\left[\frac{\partial X_i}{\partial P_i}\bigg|_{E \text{ const}} + \frac{\partial X_i}{\partial E}\frac{\partial E}{\partial P_E}\frac{\partial P_E}{\partial P_i}\right] \quad (7)$$

where E is the total quantity of energy consumed, and P_E is the price index for energy. Since the price of energy is given by the homothetic translog cost function with constant returns to scale:

[7] These elasticities are nonlinear functions of the estimated parameters, so that standard errors for their estimates cannot be calculated exactly. We therefore obtain *approximate* estimates of the standard errors under the assumption that the shares S_i are constant and equal to the means (over the estimation time bounds) of their estimated values. Under this assumption we have, asymptotically, $\text{var}(\hat{\sigma}_{ij}) = \text{var}(\hat{\gamma}_{ij})/\hat{S}_i^2\hat{S}_j^2$, $\text{var}(\hat{\sigma}_{ii}) = \text{var}(\hat{\gamma}_{ii})/\hat{S}_i^4$, $\text{var}(\hat{\eta}_{ij}) = \text{var}(\hat{\gamma}_{ij})/\hat{S}_i^2$, $\text{var}(\hat{\eta}_{ii}) = \text{var}(\hat{\gamma}_{ii})/\hat{S}_i^2$.

$$\log P_E = \alpha_0 + \sum_i \alpha_i \log P_i + \sum_i \sum_j \gamma_{ij} \log P_i \log P_j, \quad (8)$$

which implies the fuel share equations $S_i = \gamma_i + \Sigma\gamma_{ij}\log P_j$, we can show that $\eta^*_{ii} = \eta_{ii} + \eta_{EE}S_i$, where η_{EE} is the own price elasticity of aggregate energy use. Similarly, the total cross price elasticities for each fuel are given by $\eta^*_{ij} = \eta_{ij} + \eta_{EE}S_j$. The total output elasticity is given by[8]

$$\eta^*_{iQ} = \frac{d \log X_i}{d \log Q} = \frac{Q}{X_i}\frac{\partial X_i}{\partial E}\frac{\partial E}{\partial Q}. \quad (9)$$

Since the energy cost function is homothetic, this reduces to $\eta^*_{iQ} = \eta_{EQ}$, where η_{EQ} is the elasticity of energy with respect to output changes. This elasticity in turn is given by[9]

$$\eta_{EQ} = \frac{\gamma_{QE}}{S_E} + \alpha_Q + \gamma_{QQ} \log Q + \sum_{i=K,}^{L,E} \gamma_{Qi} \log P_i. \quad (10)$$

Since the cost function is not estimated directly, we assume that α_Q and γ_{QQ} are 1 and 0, respectively.

Finally, it is useful to calculate the elasticities of the average cost of production with respect to the price of energy, $\eta_{CE} = \partial\log(AC)/\partial\log P_E$, and with respect to the fuel prices, $\eta_{Ci} = \partial\log(AC)/\partial\log P_i$. From equation (3) we have

$$\begin{aligned} \eta_{CE} = \alpha_E &+ \gamma_{EE}\log P_E + \gamma_{EK}\log P_K \\ &+ \gamma_{EL}\log P_L + \gamma_{QE}\log Q. \end{aligned} \quad (11)$$

Since the energy cost function is homothetic, $\eta_{Ci} = \eta_{CE}S_i$.

Let us now review the steps involved in estimating our model of industrial energy demand. First the fuel share equations $S_i = \alpha_i + \Sigma_j\gamma_{ij}\log P_j$

[8] This assumes that the value of output is equal to the value (cost) of inputs. This would be the case under perfect competition, or under oligopoly pricing based on a fixed percentage markup over cost. Otherwise, this elasticity would be better referred to as a total cost elasticity, i.e., the percentage change in the demand for fuel i corresponding to a 1% change in the total cost of production.

[9]
$$\begin{aligned} \eta_{EQ} &= \frac{d\log E}{d\log Q} = \frac{d\log E}{d\log C} \cdot \frac{\partial\log C}{\partial\log Q} \\ &= \left[\frac{\partial\log S_E}{\partial\log Q}\frac{\partial\log Q}{\partial\log C} + 1\right]\frac{\partial\log C}{\partial\log Q} \\ &= \frac{\partial\log S_E}{\partial\log Q} + \frac{\partial\log C}{\partial\log Q} \end{aligned}$$

The two derivatives are obtained from equations (3) and (4).

are estimated, subject to the parameter restrictions $\Sigma_i \alpha_i = 1$, $\gamma_{ij} = \gamma_{ji}$, and $\Sigma_i \gamma_{ij} = \Sigma_j \gamma_{ij} = 0$. We also *test* the additional restrictions $\gamma_{ij} = 0$. The estimated values of the α_i and γ_{ij} are used in equation (8) to obtain an aggregate price index for energy. The parameter α_0 is determined so that $P_E = 1$ in the United States in 1970, and a relative energy price index is calculated for each country. Next we estimate the factor share equations (4), with $i, j = K,L,E$, using the estimated energy price index as an instrumental variable. We estimate these equations in stages, imposing and testing additional parameter restrictions at each stage.

II. Estimation of the Model

The model is estimated using pooled time-series data for a cross-section of ten countries: Canada, France, Italy, Japan, the Netherlands, Norway, Sweden, the United Kingdom, the United States, and West Germany. In estimating the model, a choice must be made as to whether the first-order and/or second-order coefficients in the share equations of each stage should be allowed to vary across countries. Allowing both sets of coefficients to vary across countries is equivalent to estimating a separate model for each country, but this would require more data than are available, and would probably result in short-run elasticity estimates. We attempted to estimate the model letting only the second-order coefficients vary across countries, but this leaves too few degrees of freedom to give satisfactory results.[10] We therefore allow the first-order coefficients to vary across countries, through the use of regional intercept dummy variables in the share equations.[11] This is also equivalent to a covariance model of the implicit error terms in those equations (i.e., error terms composed of a regional component and a total component).[12]

After estimating the model using alternative poolings, we found that a choice had to be made between pooling all ten countries together and pooling Canada and the United States separately from the European countries and Japan. We found (particularly for the energy cost function) the parameter estimates for Canada and the United States to be considerably different from those for the other countries, and this is consistent with the fact that fuel prices in Canada and the United States have been much lower throughout time. This suggests the likelihood of a different cost function for Canada and the United States, and we therefore report results here for those countries pooled separately.[13]

In estimating the model we ignore error term autocorrelation within equations, but account for error correlations across equations. We use iterative Zellner estimation, which (under the assumption of no heteroscedasticity or autocorrelation within equations) is equivalent to full-information maximum-likelihood estimation.

The share equations for the energy cost function are estimated using data over the period 1959–1973. Because of data limitations, the share equations for the total cost function are estimated using data for 1963–1973.[14] Data needed to estimate the model include cost shares and price indices for capital, labor and energy, expenditure shares and prices for the four fuels (coal, fuel oil, natural gas, and electricity), and the value of output. In computing these data series, purchasing power parity indices are used to obtain relative prices of plant and equipment in the capital input, and all energy quantities are measured in

[10] The results are reported in Pindyck (1977b).

[11] We used the chi-square statistic of equation (5) to test for homogeneity, i.e., the intercept term is the same across countries. This hypothesis is always rejected at the 1% level.

[12] A model excluding intercept dummy variables was also estimated to check whether the use of these variables might elicit a primarily short-run response. The resulting elasticity estimates are almost the same (although omission of the dummy variables gives much higher standard errors). As a second check to ensure that we are estimating long-run cost functions, the model with dummy variables was re-estimated using data at three-year intervals. Again the resulting elasticities are about the same (but with larger standard errors), so we can be fairly confident that we are picking up a primarily long-run response. These results are reported in Pindyck (1977b).

[13] Estimation results for alternative poolings are reported in Pindyck (1977b). Griffin and Gregory (1976) and Nordhaus (1975), when estimating separate demand models for the United States and several European countries, also obtained similar elasticity estimates for the European countries, but very different ones for the United States.

[14] Data were also available for 1974, a year in which energy prices rose considerably, but there is a question as to whether this additional data can be considered to have come from the same population as the 1959–1973 data, i.e., whether the 1974 expenditure shares were generated by the same *long-run* cost function. We found that the 1974 data should *not* be considered to belong to the same population as the earlier data. Tests and elasticity estimates resulting from inclusion of the 1974 data are reported in Pindyck (1977b). (As expected, estimated elasticities are smaller when the 1974 data are used, since it represents a short-run cost function.)

"gross" rather than "net" terms.[15] The sources of data and methods of constructing series for expenditure shares and prices are described in an appendix available from the author on request.

III. Results

We begin with the estimation of the energy cost function. Estimated parameters for the share equations are shown in table 1. The R^2s for all of the share equations are high, but a good deal of this explanation can be attributed to the country intercept terms.[16] On the other hand, for both sets of estimates, 13 of the 16 second-order coefficients are statistically significant.

[15] That is, we do not adjust fuel quantities or prices by thermal efficiencies of utilization. There are two reasons for this: first, there are no good estimates of thermal efficiencies available, and second, there are other "economic" efficiency measures that could be equally important in affecting consumer demand. For a further discussion of this issue, see Pindyck (1977a).

[16] When an energy cost function for all ten countries was estimated without country dummy variables, the R^2s for the coal, oil and gas share equations were 0.045, 0.400, and 0.227, respectively.

The partial fuel price elasticities corresponding to these parameter estimates are shown in table 2. We see that except for electricity the elasticities are generally large in magnitude. Own price elasticities for coal range from −1 in France and West Germany to about −2 in Norway, Canada, and the United States (where coal has had a smaller share of industrial energy consumption). Own price elasticities for natural gas are between −1.3 and −2.3 in the European countries and Japan, but −0.33 in Canada and −0.52 in the United States, which is reasonable given that natural gas prices have been much lower in Canada and the United States. On the other hand, Canada and the United States have the largest own price elasticities for oil, even though they had relatively low prices. We can only explain this on the basis of a greater availability of alternative fuels at low prices (notably natural gas), and the availability of "interruptible" contracts for natural gas, so that producers choose technologies allowing for greater interfuel substitution possibility. Finally, the own price elasticities for electricity are all quite small. This is

TABLE 1.—PARAMETER ESTIMATES: ENERGY COST FUNCTION

	U.S. & Canada	Europe & Japan		U.S. & Canada	Europe & Japan
$\alpha_1 D_1$	.2335 (.0218)		$\alpha_4 D_1$	.2569 (.0233)	
$\alpha_1 D_2$		.2438 (.0268)	$\alpha_4 D_2$		.1821 (.0223)
$\alpha_1 D_3$		.2379 (.0231)	$\alpha_4 D_3$		.2104 (.0226)
$\alpha_1 D_4$		.2571 (.0314)	$\alpha_4 D_4$		.2218 (.0271)
$\alpha_1 D_5$		.1486 (.0368)	$\alpha_4 D_5$		.2067 (.0334)
$\alpha_1 D_6$		−.0613 (.0213)	$\alpha_4 D_6$		.6408 (.0116)
$\alpha_1 D_7$		.1221 (.0280)	$\alpha_4 D_7$		.3577 (.0116)
$\alpha_1 D_8$		.1768 (.0346)	$\alpha_4 D_8$		.1833 (.0266)
$\alpha_1 D_9$	.2555 (.0382)		$\alpha_4 D_9$	.2238 (.0386)	
$\alpha_1 D_{10}$		.2864 (.0286)	$\alpha_4 D_{10}$		.2268 (.0266)
$\alpha_2 D_1$	.2099 (.0162)		β_{11}	−.1017 (.0406)	−.1039 (.0293)
$\alpha_2 D_2$		.3807 (.0228)	β_{12}	.0739 (.0256)	.0093 (.0210)
$\alpha_2 D_3$		.4052 (.0212)	β_{13}	.1195 (.0201)	.1184 (.0112)
$\alpha_2 D_4$		.3573 (.0255)	β_{14}	−.0917 (.0164)	−.0237 (.0142)
$\alpha_2 D_5$		.4137 (.0301)	β_{21}	.0739 (.0256)	.0093 (.0210)
$\alpha_2 D_6$		.3317 (.0178)	β_{22}	−.0152 (.0233)	.1063 (.0210)
$\alpha_2 D_7$		.3839 (.0226)	β_{23}	−.0754 (.0128)	−.0240 (.0105)
$\alpha_2 D_8$		.4037 (.0261)	β_{24}	.0167 (.0151)	−.0916 (.0110)
$\alpha_2 D_9$	.0501 (.0305)		β_{31}	.1195 (.0201)	.1184 (.0112)
$\alpha_2 D_{10}$		.3513 (.0244)	β_{32}	−.0754 (.0128)	−.0240 (.0105)
$\alpha_3 D_1$	.2997 (.0220)		β_{33}	.0557 (.0177)	−.0340 (.0087)
$\alpha_3 D_2$		.1934 (.0147)	β_{34}	−.0998 (.0141)	−.0604 (.0063)
$\alpha_3 D_3$		.1466 (.0125)	β_{41}	−.0917 (.0164)	−.0237 (.0142)
$\alpha_3 D_4$		.1638 (.0153)	β_{42}	.0167 (.0151)	−.0916 (.0110)
$\alpha_3 D_5$		.2311 (.0179)	β_{43}	.0998 (.0141)	−.0604 (.0063)
$\alpha_3 D_6$		.0888 (.0140)	β_{44}	.1748 (.0176)	.1758 (.0126)
$\alpha_3 D_7$		.1362 (.0151)	R^2		
$\alpha_3 D_8$		.2362 (.0171)	Eq. 1	.260	.746
$\alpha_3 D_9$	.4705 (.0325)		Eq. 2	.959	.471
$\alpha_3 D_{10}$		.1356 (.0145)	Eq. 3	.955	.633

Note Standard errors are in parentheses. Fuels are numbered 1 = coal, 2 = oil, 3 = gas, 4 = electricity Country intercepts are numbered 1 = Canada, 2 = France, 3 = Italy, 4 = Japan, 5 = Netherlands, 6 = Norway, 7 = Sweden, 8 = U K, 9 = U S A., 10 = W. Germany

TABLE 2.—PARTIAL FUEL PRICE ELASTICITIES

Elasticity	CAN	FRAN	ITAL	JAP	NETH	NOR	SWED	UK	USA	WGER
η_{11}	−1.80	−1.04	−1.49	−1.32	−1.67	−2.08	−1.26	−1.12	−2.17	−1.09
	(0.36)	(0.10)	(0.18)	(0.15)	(0.22)	(0.33)	(0.13)	(0.11)	(0.50)	(0.10)
η_{12}	0.90	0.20	0.27	0.21	0.21	0.37	0.24	0.21	0.99	0.15
	(0.23)	(0.07)	(0.13)	(0.11)	(0.16)	(0.24)	(0.10)	(0.08)	(0.31)	(0.07)
η_{13}	1.17	0.46	0.83	0.65	0.98	1.34	0.55	0.52	1.66	0.43
	(0.18)	(0.04)	(0.07)	(0.06)	(0.09)	(0.13)	(0.05)	(0.04)	(0.25)	(0.04)
η_{14}	−0.28	0.39	0.39	0.45	0.48	0.37	0.46	0.39	−0.48	0.49
	(0.15)	(0.05)	(0.09)	(0.07)	(0.11)	(0.16)	(0.07)	(0.05)	(0.20)	(0.05)
η_{21}	0.41	0.36	0.20	0.26	0.20	0.12	0.27	0.32	0.97	0.37
	(0.10)	(0.12)	(0.10)	(0.13)	(0.15)	(0.08)	(0.10)	(0.12)	(0.31)	(0.18)
η_{22}	−0.81	−0.20	−0.29	−0.20	−0.11	−0.34	−0.27	−0.22	−1.10	0.03
	(0.09)	(0.12)	(0.10)	(0.13)	(0.15)	(0.08)	(0.10)	(0.12)	(0.28)	(0.18)
η_{23}	−0.21	−0.08	−0.03	−0.08	−0.10	−0.09	−0.11	−0.06	−0.72	−0.18
	(0.05)	(0.06)	(0.05)	(0.06)	(0.07)	(0.04)	(0.05)	(0.06)	(0.15)	(0.09)
η_{24}	0.61	−0.08	0.11	0.02	0.01	0.30	0.12	−0.04	0.85	−0.22
	(0.06)	(0.07)	(0.05)	(0.07)	(0.08)	(0.04)	(0.05)	(0.06)	(0.18)	(0.10)
η_{31}	1.35	2.21	1.52	2.12	1.84			1.86	0.72	4.98
	(0.21)	(0.18)	(0.13)	(0.18)	(0.16)	[a]	[a]	(0.15)	(0.11)	(0.44)
η_{32}	−0.53	−0.22	−0.06	−0.22	−0.21			−0.15	−0.32	−0.83
	(0.13)	(0.17)	(0.12)	(0.17)	(0.15)	[a]	[a]	(0.14)	(0.07)	(0.42)
η_{33}	−0.33	−1.49	−1.30	−1.49	−1.42			−1.38	−0.52	−2.31
	(0.18)	(0.14)	(0.10)	(0.14)	(0.13)	[a]	[a]	(0.12)	(0.09)	(0.34)
η_{34}	−0.49	−0.51	−0.15	−0.41	−0.22			−0.33	0.12	−1.82
	(0.15)	(0.10)	(0.07)	(0.10)	(0.09)	[a]	[a]	(0.09)	(0.08)	(0.25)
η_{41}	−0.06	0.25	0.12	0.16	0.09	0.05	0.18	0.22	−0.06	0.25
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
η_{42}	0.28	−0.03	0.04	0.01	0.00	0.12	0.04	−0.01	0.11	−0.05
	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
η_{43}	−0.09	−0.07	−0.02	−0.04	−0.02	−0.09	−0.10	−0.05	−0.03	−0.08
	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)
η_{44}	−0.14	−0.16	−0.13	−0.12	−0.07	−0.08	−0.12	−0.15	−0.08	−0.12
	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)

Note 1 = coal, 2 = oil, 3 = gas, 4 = electricity.
[a] Almost no natural gas is consumed in the industrial sectors of Norway and Sweden, so that these elasticities are meaningless

not surprising; since electricity is a much more expensive fuel on a thermal basis, it should be used only where there is no possibility of using an alternative fuel.

The energy cost function is now used to generate the aggregate price index for energy. We choose α_0, the unobservable parameter in the cost function, so that $P_E = 1.0$ in the United States in 1970, and use the fuel price data to generate the relative energy price index over time for each country. The resulting index serves as the instrumental variable for the price of energy in the estimation of the total cost function.

Parameter estimates for the total cost function are shown in table 3. Again, although much of the explanation can be attributed to the country intercept terms (so that the R^2s are small for the United States and Canada), most of the second-order parameters are significant. We also estimate a homothetic version of this model in order to test the hypothesis of homotheticity. For Europe and Japan the test statistic is 8.58, and this is significant at the 2.5% level, so that homotheticity cannot be accepted here. For Canada and the United States the test statistic is 1.25, which is not significant at the 10% level. However, if the country dummy variable is eliminated, the same test statistic is 32.4, which is significant at the 1% level. Since a test for constancy of the first-order terms in the non-homothetic model of Canada and the United States gives a test statistic of 2.35, which is not significant at the 10% level, we consider the test for homotheticity to be inconclusive, and retain the non-homothetic model.[17]

The elasticities of substitution and the own price elasticities for capital, labor and energy implied by this cost function are shown in table 4. We see that the elasticity of substitution for energy and capital (σ_{KE}) is positive, so that these inputs are *substitutes*, and not complements as

[17] We also tested the hypothesis that the γ_{ij} are all zero, i.e., that the corresponding production function is Cobb-Douglas. The test statistics are significant at the 1% level, so that this hypothesis can be rejected.

TABLE 3.—PARAMETER ESTIMATES: TOTAL COST FUNCTION

	U.S. & Canada	Europe & Japan		U.S. & Canada	Europe & Japan
$\alpha_K D_1$	0.8375 (.3930)		$\alpha_E D_1$	−.0295 (.9794)	
$\alpha_K D_2$		.0649 (.2584)	$\alpha_E D_2$		.1791 (.0447)
$\alpha_K D_3$		.0432 (.2356)	$\alpha_E D_3$		.1891 (.0400)
$\alpha_K D_4$		.0969 (.2890)	$\alpha_E D_4$		.2097 (.0500)
$\alpha_K D_5$		.0919 (.1899)	$\alpha_E D_5$		.1539 (.0361)
$\alpha_K D_6$		.1748 (.1465)	$\alpha_E D_6$		.1375 (.0245)
$\alpha_K D_7$		.0829 (.1898)	$\alpha_E D_7$		.1650 (.0332)
$\alpha_K D_8$		−.1353 (.2575)	$\alpha_E D_8$		.1908 (.0448)
$\alpha_K D_9$	1.0104 (.5869)		$\alpha_E D_9$	−.0813 (.9003)	
$\alpha_K D_{10}$		.1438 (.2792)	$\alpha_E D_{10}$		.1862 (.0480)
$\alpha_L D_1$	0.1929 (.3614)		γ_{KK}	−1068 (.0795)	.0732 (.0270)
$\alpha_L D_2$		.7559 (.2479)	γ_{KL}	.0957 (.0728)	−.0628 (.0254)
$\alpha_L D_3$		.7677 (.2261)	γ_{KE}	.0111 (.0083)	−.0104 (.0032)
$\alpha_L D_4$		.6934 (.2773)	γ_{LK}	.0957 (.0728)	−.0628 (.0254)
$\alpha_L D_5$		.7542 (.1828)	γ_{LL}	−.0808 (.0673)	.0585 (.0243)
$\alpha_L D_6$		.6880 (.1405)	γ_{LE}	−.0149 (.0087)	.0043 (.0039)
$\alpha_L D_7$		.7521 (.1820)	γ_{EK}	.0111 (.0083)	−.0104 (.0032)
$\alpha_L D_8$		.9448 (.2471)	γ_{EL}	−.0149 (.0087)	.0043 (.0039)
$\alpha_L D_9$	0.0710 (.5393)		γ_{EE}	.0038 (.0042)	.0061 (.0031)
$\alpha_L D_{10}$		.6701 (.2679)	γ_{QK}	−.0648 (.0727)	.0539 (.0380)
			γ_{QL}	.0513 (.0364)	−.0335 (.0364)
			γ_{QE}	.0135 (.0091)	−.0203 (.0065)
			Eqn.	R^2	R^2
			1	.259	.880
			2	.165	.885

Note: Standard errors are in parentheses, country intercepts are numbered: 1 = Canada, 2 = France, 3 = Italy, 4 = Japan, 5 = Netherlands, 6 = Norway, 7 = Sweden, 8 = U.K., 9 = U.S.A., 10 = W. Germany

earlier studies had indicated. Also, note that the largest values of σ_{KE} are for the United States and Canada, and these countries have smaller elasticities of substitution for *labor* and energy.[18] We find, as expected, that labor and energy are substitutes (with elasticities of substitution greater than 1 in Europe and Japan) and that capital and labor are substitutes. The own price elasticities of demand for capital and labor are around −0.3 to −0.7, which is in agreement with most earlier work. We find the own price elasticity for energy, however, to be about −0.8, whereas the estimates of others have generally been around −0.4 to −0.5. Most earlier work, however, was based on data for a single country, which probably elicits a *short-run* elasticity.[19]

[18] Berndt and Wood (1975), for example, found strong complementarity between energy and capital. Griffin and Gregory (1976), using pooled international data at four-year intervals, obtain results similar to ours.

[19] Our estimate is close to that found by Griffin and Gregory (1976), who also used international data.

TABLE 4.—PRICE AND SUBSTITUTION ELASTICITIES FOR CAPITAL, LABOR, AND ENERGY

	CAN	FRAN	ITAL	JAP	NETH	NOR	SWE	UK	USA	WGER
σ_{KK}	−1.75	−0.79	−0.98	−0.59	−1.16	−0.97	−1.20	−1.78	−1.66	−0.49
	(0.39)	(0.13)	(0.16)	(0.09)	(0.19)	(0.15)	(0.20)	(0.41)	(0.37)	(0.08)
σ_{KL}	1.43	0.72	0.70	0.70	0.70	0.71	0.69	0.64	1.41	0.71
	(0.32)	(0.11)	(0.12)	(0.12)	(0.12)	(0.12)	(0.12)	(0.15)	(0.31)	(0.12)
σ_{KE}	1.48	0.56	0.67	0.74	0.59	0.59	0.63	0.36	1.77	0.66
	(0.36)	(0.13)	(0.10)	(0.08)	(0.13)	(0.13)	(0.11)	(0.19)	(0.58)	(0.10)
σ_{LL}	−1.33	−0.83	−0.73	−1.22	−0.60	−0.70	−0.60	−0.34	−1.29	−1.30
	(0.27)	(0.11)	(0.09)	(0.17)	(0.08)	(0.09)	(0.08)	(0.05)	(0.26)	(0.18)
σ_{LE}	0.42	1.17	1.11	1.15	1.11	1.14	1.10	1.10	0.05	1.23
	(0.35)	(0.16)	(0.10)	(0.14)	(0.10)	(0.12)	(0.09)	(0.09)	(0.56)	(0.21)
σ_{EE}	−16.96	−16.45	−11.06	−11.28	−12.39	−13.92	−10.88	−13.25	−27.21	−16.05
	(1.58)	(1.22)	(0.54)	(0.56)	(0.67)	(0.86)	(0.52)	(0.78)	(4.35)	(1.16)
η_{KK}	−0.78	−0.37	−0.41	−0.32	−0.43	−0.41	−0.43	−0.46	−0.71	−0.29
	(0.18)	(0.06)	(0.06)	(0.05)	(0.07)	(0.06)	(0.07)	(0.10)	(0.17)	(0.05)
η_{LL}	−0.66	−0.40	−0 37	−0.46	−0.34	−0.37	−0.33	−0.23	−0.65	−0.47
	(0.14)	(0.05)	(0.05)	(0.06)	(0.04)	(0.05)	(0.04)	(0.04)	(0.13)	(0.07)
η_{EE}	−0.87	−0.83	−0.84	−0.84	−0.84	−0.84	−0.84	−0.84	−0.85	−0.85
	(0.08)	(0.06)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.05)	(0.14)	(0.06)

Finally, our estimates of the cross price elasticities of energy and capital (not reported here) indicate that over the long run a doubling in the price of energy should result in increases in the demand for capital ranging from 2% to 8% across countries, and similar increases in the demand for labor, as substitution away from energy takes place.

We next calculate the index of scale economies (SCE) introduced by Christensen and Greene (1976), and the elasticity of energy demand with respect to output changes.[20] (The indices and elasticities for each country are calculated at the point of means.) Note in table 5 that the index of scale economies is insignificantly different from zero for each country, so that the aggregate cost functions exhibit nearly constant returns to scale. The output elasticity of energy demand, however, is significantly less than unity. Thus, even if energy prices remain constant relative to other prices, as output increases there will be substitution away from energy.

Total price elasticities for individual fuel demands are shown in table 6. Note that these total own price elasticities are larger in magnitude than the corresponding partial elasticities, since they account for decreased use of energy as well as interfuel substitution. We find that coal has the

TABLE 5.—INDEX OF SCALE ECONOMIES AND OUTPUT ELASTICITY OF ENERGY DEMAND

Country	SCE	η_{EQ}
CAN	.0015 (.0080)	.785 (.108)
FRAN	.0086 (.0383)	.783 (.113)
ITAL	.0032 (.0335)	.855 (.078)
JAP	.0105 (.0487)	.849 (.087)
NETH	−.0124 (.0170)	.818 (.093)
NOR	.0056 (.0172)	.807 (.097)
SWE	.0019 (.0205)	.864 (.070)
UK	.0041 (.0327)	.778 (.113)
USA	.0003 (.0037)	.624 (.188)
WGER	.0012 (.0316)	.761 (.122)

Note: Standard errors are computed based on constancy of shares and prices at their mean values The standard error of SCE is thus computed from

$$\text{var(SCE)} = \sum_{i=K}^{L,M} (\log P_i)^2 \ \text{var}(\gamma_{Qi}) + \sum_{i \neq j} \log P_i \log P_j \text{covar}(\gamma_{Qi}\gamma_{Qj})$$

and the standard error of η_{EQ} is computed from

$$\text{var}(\eta_{EQ}) = \text{var(SCE)} + [2 \log P_E/S_E + 1/S_E^2]\text{var}(\gamma_{EQ}) + (2/S_E)\log P_K \text{covar}(\gamma_{QE}\gamma_{QK}) + (2/S_E)\log P_l \text{covar}(\gamma_{QE}\gamma_{Ql}).$$

largest own price elasticities, ranging from −1.29 to −2.24. For Europe and Japan, own price elasticities for natural gas are large (−1.37 to −2.34), while those for oil are small (−0.6 to −0.56). We attribute this to the fact that for two countries (Netherlands and W. Germany) as oil and gas prices fell, there was a large increase in the share of natural gas (from almost zero) as supplies became available for the first time. This might have tended to bias the natural gas elasticities upwards. It is more difficult to explain the low oil price elasticities; oil prices on a thermal basis were generally the lowest of any fuel, but oil did not gain a dominant share in Europe.

Elasticities of average cost of output with respect to the price of energy and the prices of the individual fuels are shown in table 7. These elas-

[20] The index of scale economies is defined as SCE = $1 - \partial \log C/\partial \log Q = 1 - (\alpha_Q + \gamma_{QQ} \log Q + \Sigma_i \gamma_{Qi} \log P_i)$. Note that if SCE is positive (negative), there is increasing (decreasing) returns to scale. This is a useful index, and has a natural interpretation in percentage terms. However, it can be computed only if α_Q and γ_{QQ} are known (which means estimating the cost function), or are assumed to be 1 and 0, respectively.

TABLE 6.—TOTAL FUEL PRICE ELASTICITIES

Elasticity	CAN	FRAN	ITAL	JAP	NETH	NOR	SWE	UK	USA	WGER
η_{11}	−1.89	−1.29	−1.63	−1.49	−1.78	−2.15	−1.44	−1.35	−2.24	−1.31
η_{12}	0.69	0.06	0.09	0.07	0.09	0.15	0.07	0.06	0.92	0.05
η_{13}	1.08	0.40	0.76	0.60	0.92	1.33	0.54	0.45	1.50	0.41
η_{14}	−0.75	0.0	−0.06	−0.03	−0.08	−0.16	−0.02	−0.02	−1.03	0.01
η_{21}	0.31	0.11	0.07	0.09	0.09	0.05	0.08	0.10	0.90	0.13
η_{22}	−1.03	−0.34	−0.46	−0.35	−0.22	−0.56	−0.44	−0.37	−1.17	−0.06
η_{23}	−0.29	−0.13	−0.10	−0.13	−0.16	−0.09	−0.12	−0.12	−0.88	−0.20
η_{24}	0.14	−0.46	−0.35	−0.46	−0.54	−0.24	−0.37	−0.44	0.30	−0.70
η_{31}	1.25	1.96	1.39	1.95	1.73	a	a	1.64	0.65	4.73
η_{32}	−0.75	−0.36	−0.24	−0.36	−0.33	a	a	−0.30	−0.38	−0.93
η_{33}	−0.41	−1.54	−1.37	−1.54	−1.48	a	a	−1.44	−0.67	−2.34
η_{34}	−0.96	−0.89	−0.61	−0.89	−0.77	a	a	−0.74	−0.43	−2.29
η_{41}	−0.15	0.0	−0.01	−0.01	−0.02	−0.02	−0.01	−0.01	−0.13	0.01
η_{42}	0.06	−0.17	−0.14	−0.13	−0.11	−0.10	−0.13	−0.16	0.04	−0.14
η_{43}	−0.17	−0.12	−0.10	−0.09	−0.08	−0.09	−0.10	−0.11	−0.13	−0.10
η_{44}	−0.61	−0.54	−0.59	−0.60	−0.63	−0.62	−0.60	−0.56	−0.63	−0.59

Note 1 = solid, 2 = liquid, 3 = gas, 4 = electricity

[a] Almost no natural gas is consumed in the industrial sectors of Norway and Sweden, so that these elasticities are meaningless

TABLE 7.—ELASTICITY OF AVERAGE COST OF OUTPUT WITH RESPECT TO PRICE OF ENERGY AND FUELS

Elasticity	Year	CAN	FRAN	ITAL	JAP	NETH	NOR	SWE	UK	USA	WGER
η_{CE}	1963	.045	.053	.076	.073	.069	.062	.078	.065	.029	.051
	1972	.050	.046	.067	.063	.060	.062	.066	.059	.032	.043
η_{C1}	1963	.006	.017	.009	.012	.010	.004	.015	.019	.003	.014
	1972	.004	.008	.010	.007	.004	.006	.010	.008	.002	.005
η_{C2}	1963	.011	.008	.018	.012	.013	.016	.012	.010	.002	.007
	1972	.013	.012	.014	.014	.004	.016	.015	.016	.003	.007
η_{C3}	1963	.004	.005	.006	.006	.000	.000	.001	.005	.005	.001
	1972	.007	.004	.007	.002	.011	.000	.000	.005	.006	.003
η_{C4}	1963	.028	.023	.043	.044	.045	.044	.050	.030	.019	.030
	1972	.026	.023	.037	.040	.041	.040	.041	.030	.020	.028

Note: E = Energy, 1 = solid fuel, 2 = liquid fuel, 3 = gas, 4 = electricity

ticities provide information about the inflationary impact of an energy price rise, assuming the level of output stays fixed.[21] Note that in the United States a 10% increase in the cost of energy would result in about a 0.3% increase in the cost of output, whereas for Italy, Japan, and Sweden the cost of output would rise about 0.7%.

[21] In comparing these elasticities across countries, remember that they are dependent on the shares of energy, and the fuel shares. Thus Italy, Japan and Sweden have the largest values of η_{CE} in part because they have the largest shares of energy in the cost of output.

IV. Comparison of Results with Other Studies

In table 8 we present a survey of recent estimates by others of industrial energy demand

TABLE 8.—ALTERNATIVE ESTIMATES OF INDUSTRIAL DEMAND ELASTICITIES

Elasticity	Country	Estimate	Source
Factor Inputs—Elasticities of Substitution	USA	$\sigma_{KL} = 1.01$, $\sigma_{KE} = -3.25$, $\sigma_{LE} = 0.64$	(a)
	USA (2-digit industries)	$\sigma_{KE} = -1.03$ to 2.02, $\sigma_{LE} = .48$ to 2.88 (prod. workers), $\sigma_{LE} = -2.02$ to 5.59 (non-prod. workers)	(b)
	CAN	$\sigma_{KL} = 0.72$, $\sigma_{KE} = 0.42$, $\sigma_{LE} = 1.70$	(c)
	CAN	$\sigma_{KL} = 5.46$, $\sigma_{KE} = -11.91$, $\sigma_{LE} = 4.89$	(d)
	NETH	$\sigma_{KL} = 1.09$, $\sigma_{KE} = -4.41$, $\sigma_{LE} = 2.30$	(e)
	9 industrialized countries	$\sigma_{KL} = 0.06$ to 0.52, $\sigma_{KE} = 1.02$ to 1.07, $\sigma_{LE} = 0.72$ to 0.87.	(f)
Factor Inputs—Price Elasticities	USA	$\eta_{KK} = -0.44$, $\eta_{LL} = -0.45$, $\eta_{EE} = -0.49$, $\eta_{KE} = -0.15$, $\eta_{LE} = 0.03$.	(a)
	USA (2-digit industries)	$\eta_{KK} = -0.67$ to -1.16, $\eta_{LL} = -0.28$ to -1.55, $\eta_{EE} = -0.66$ to -2.56	(b)
	CAN	$\eta_{KK} = -0.79$, $\eta_{LL} = -0.45$, $\eta_{EE} = -0.36$	(c)
	CAN	$\eta_{KK} = -0.31$, $\eta_{LL} = -0.77$, $\eta_{EE} = -0.59$	(d)
	CAN	$\eta_{KK} = -0.76$, $\eta_{LL} = -0.49$, $\eta_{EE} = -0.49$, $\eta_{KE} = -0.05$, $\eta_{LE} = 0.55$.	(g)
	NETH	$\eta_{KK} = -0.42$, $\eta_{LL} = -0.46$, $\eta_{EE} = -0.29$.	(e)
	9 industrialized countries	$\eta_{KK} = -0.18$ to -0.38, $\eta_{LL} = -0.12$ to -0.27, $\eta_{EE} = -0.79$ to -0.80, $\eta_{KE} \simeq 0.13$, $\eta_{LE} \simeq 0.11$	(f)
	6-country composite	$\eta_{EE} = -0.30$	(h)
Fuels—Own Price Elasticities, Partial	USA	elec: -0.66, oil: -2.75, gas: -1.30, coal: -1.46	(i)
	USA	elec., S.R.: -0.14, elec., L.R.: -1.20	(j)
	USA	elec., S.R.: -0.06, elec., S.R.: -0.52	(k)
	USA	elec: -0.92, oil: -2.82, gas: -1.47, coal: -1.52	(i)
Own Price Elasticities, Total	CAN	elec: -0.74, oil: -1.30, gas: -1.30, coal: -0.48	(g)

Sources (a) Berndt and Wood (1975), (b) Halvorsen and Ford (1978); (c) Fuss and Waverman (1975), translog, (d) Fuss and Waverman (1975), generalized Leontief, (e) Magnus (1979), (f) Griffin and Gregory (1976), (g) Fuss (1977), (h) Nordhaus (1975), (i) Halvorsen (1976), (j) Mount, Chapman, and Tyrrell (1973), (k) Griffin (1974) elasticities. Note that there is mixed evidence on the substitutability of energy and capital. Berndt and Wood (1975), Fuss (1977), and Magnus (1975) find energy and capital to be strong complements, but they worked with data for a single country, and might have estimated a short-run cost or production function. Halvorsen and Ford (1978) and Fuss and Waverman (1975) obtain mixed results on energy-capital substitutability, which depended on the particular disaggregated industry or the particular form of the cost function. Only Griffin and Gregory (1976) find strong evidence of capital-energy substitutability, and their estimate of the Allen elasticity of substitution is close to ours (1.01 compared to about 0.8). As for elasticities of substitution between other factors, our results are close to Griffin and Gregory for labor and energy, but we find greater substitution of capital and labor.

There are two ways to reconcile the capital-energy substitutability found by Griffin and Gregory and by this study with the complementarity found by Berndt and Wood. First, the use of international data is more likely to permit estimation of the long-run cost function, and we might expect to observe complementarity in the shorter run and substitutability in the long run. Second, we must keep in mind that both Griffin and Gregory and this study use a three-factor total cost function, while Berndt and Wood work with four factors. As recently pointed out by Berndt and Wood (1977), complementarity between two factors in a four-dimensional production space can be consistent with substitutability between the same factors in a three-dimensional subspace.

The own price elasticity of aggregate energy use is an important parameter to any energy policy debate. Our estimate (about −0.8), together with that of Griffin and Gregory (also −0.8), is larger than most other estimates, which fall in the range of −0.3 to −0.6, and that are larger still than the "consensus" estimates often used for policy analysis in the United States (−0.2 to −0.3). Again, most other estimates are based on time series data for a single country, and may be short run. Our estimates of the output elasticity of energy use (0.7 to 0.8) are smaller than most "consensus" estimates (about 1), and are due to the non-homotheticity of the estimated aggregate cost function.

It is more difficult to find a consensus on partial and total fuel price elasticities. Although most would agree that electricity demand is less elastic than the demands for other fuels, partial long-run elasticities for the United States range from −0.5 to −1.2. Our study finds electricity demand to be even less elastic; we found partial own price elasticities ranging from −0.08 to −0.16. Our *total* own price elasticity estimates, however, are closer to the estimates of others (largely because of our higher estimate of the own price elasticity of energy). We find this elasticity to range from −0.54 to −0.63, where Halvorsen (1976) obtained an estimate of −0.92 and Fuss (1977) −0.74. Our own price elasticity estimate (total) for oil is also well below the estimates of others; −0.22 to −1.17 as compared to Halvorsen's estimate of −2.82 and Fuss's of −1.30. An explanation for this discrepancy will probably require further work. There is less disagreement over the elasticities for coal and natural gas. Our estimates of the total own price elasticities for coal (−1.29 to −2.24) and natural gas (−0.41 to −2.34) are generally in line with other estimates.

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